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Expansion of a Finite Mass of Gas into Vacuum

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SOME years ago, the authors studied the problem of the expansion into vacuum of a finite mass of gas initially at rest and in a uniform state, under the assumptions that the gas is perfect and inviscid. The results for the case of plane flow appear in Ref. 1, and those for cylindrical and spherical flow are unpublished. From time to time we have, upon request, privately communicated some of the results, together with assurances of the eventual publication of the results in their entirety. However, the continued passage of time has tended to diminish the conviction of these assurances. It therefore seems advisable, at this time, at least to summarize the contents of Ref. 1, and to provide perhaps the most useful result of our unpublished calculations.

We considered the three cases of plane, cylindrical, and spherical symmetry. The flow in the plane case can be described[2] as the interaction of two expansion fans, or simple waves, centered about the edges ($\pm x_0$) of the initial mass of gas. The x, t plane, or the flow at any time t, is thus divided into two essential regimes, a simple wave and an interaction region. In the cylindrical and spherical cases, the regions of the r, t plane are essentially the same, although pure simple waves no longer exist. In all cases, the flow at any point consists first of the outward motion caused by the expansion fan from the nearest boundary, and then a weakening of this process by the arrival of the expansion fan from the other boundary (plane case) or a reflection from the center (cylindrical and spherical cases).

In the case of plane flow, we constructed, by standard methods,[2] an exact analytic solution valid for all t. The cylindrical and spherical cases, however, are not amenable to such a treatment. However, since all of the gas eventually enters the interaction region, a solution, asymptotic for large t, in the interaction region provides useful information about the flow. It is precisely such a solution that we constructed by similarity methods.

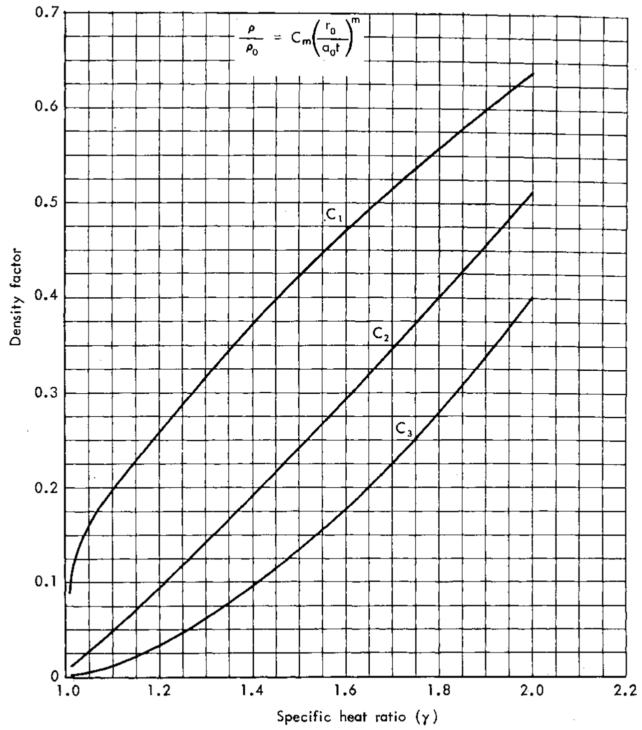

Fig. 1 Asymptotic dependence of density on specific-heat ratio.

The principal results of the analysis are 1) the flow velocity $u(r, t)$ is, in all three cases,

$$u(r, t) \cong r/t \tag{1}$$

where r is the distance from the origin; and 2)

$$\rho(r, t)/\rho_0 \cong C_m(\gamma)\ (r_0/a_0 t)^m \tag{2}$$

$m = 1$ plane case $= 2$ cylindrical case $= 3$ spherical case

where ρ_0 is the density, a_0 the sound speed, and r_0 the radius of the initially-uniform gas. The constant in Eq. (2) depends only on m and on the gas constant γ. This solution is asymptotically valid in the region of the r, t plane not too close to the expansion front.

As we mentioned previously, in the case of plane flow an exact analytic solution of the problem can be obtained. By examining this solution in the limit of large t, we were able not only to verify that the assumed similarity solution is, in fact, the asymptotic flow, but we obtained at the same time an analytic expression for the constant $C_1(\gamma)$, viz.,

$$C_1(\gamma) = 2^{\lambda-1}[\Gamma(2\nu - 1)]^{\lambda}/[\Gamma(\nu)]^{2\lambda} \tag{3}$$

where $\lambda = (\gamma - 1)/2$ and $\nu = \frac{1}{2}(\gamma + 1)/(\gamma - 1)$. In Eq. (3), Γ denotes the usual gamma function. In the cylindrical and spherical cases, the asymptotic validity of the similarity solution was established by a numerical integration of the equations of motion. Integration of the equations for different values of γ served to determine the constants C_2 and C_3 as functions of γ. The dependence of C_m on γ for the various cases is shown in Fig. 1.

The results of this study have practical application as an approximation to the expansion of a finite mass of gas into a gas at much lower pressure and density. In this case, a shock wave precedes the expanding gas, but in the limit of ambient vacuum, the effect of the shock wave disappears from the problem. Another application follows from the use

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of the unsteady analogy of hypersonic small-deflection theory whereby the cylindrical unsteady flow becomes analogous to the flow of a high Mach number jet to vacuum.

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Experimentally Determined Reynolds Analogy Factors for Flat Plates

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IN fluid mechanics, the analogy between skin friction and heat-transfer coefficients is remarkable in its simplicity and wide range of usefulness. The original analogy formulated by Reynolds in 1874 is now usually written

$$C_h = C_f/2 \tag{1}$$

and is based on the assumption that momentum and heat are transferred by similar processes. Because such a relationship depends upon the relative diffusion of momentum and heat through the boundary layer, the effect of the Prandtl number must be taken into account in order to provide general results. For a smooth, flat surface, it is common to write

$$C_h = (1/S)(C_f/2) \tag{2}$$

where S is designated the Reynolds analogy factor and is a strong function of the Prandtl number. For the low-speed, turbulent-flow case, Colburn has shown that the modified Reynolds analogy

$$C_h = (1/P_r^{2/3})(C_f/2) \tag{3}$$

properly predicts the variation of S with P_r; this relation has been found to give good results for air with Reynolds numbers of less than 10^6.[1] The expression, of course, reduces to the simple Reynolds analogy for $P_r = 1$. Several modifications[1-4] to the Reynolds analogy have been devised to account for the effects of compressibility and dissipation in high-speed flow.

Extensive theoretical and experimental studies have been reported on the Reynolds analogy for various surfaces and flow conditions, but there appears to be little experimental information available for the case of compressible turbulent flow over rough surfaces. The purpose of this note is to present some recent results from the simultaneous measurement of local skin-friction and heat-transfer rate at adjacent positions on a flat-plate model. The measurements were made with both smooth and rough plate surfaces, as part of a general study of the influence of roughness of skin friction and heat transfer in turbulent, compressible flows.

Experimental Apparatus

An intermittent flow wind tunnel having a 6 × 7-in. test section and fixed nozzle blocks was employed for the experiments. The nominal Mach number was 4.93, the stilling-chamber pressure was 255 psia, and a stagnation temperature range of 620–1100°R was employed. The corresponding range of Reynolds number was 0.5 to 1.5 × 10^7/ft. The nominal plate temperature was 555°R, resulting in a range of wall-to-freestream temperature ratio of 2.9 to 5.2. A more complete description of the tunnel and its associated equipment is given in Ref. 5.

The basic flat-plate model was 6 in. wide, 1 in. thick, and 17.5 in. long. A 15° wedge angle was used for the plate nose. The plate body was constructed of copper with integral cooling passages to maintain a constant temperature during each run. The model spanned a test section with the flat test surface facing downward. A slug-type calorimeter and a floating-element skin-friction balance, both employing 1-in.-diam disks, were located 12.5 in. aft of the leading edge. The 20-g, 0.150-in.-thick copper calorimeter was insulated from the plate by an air space, except where it was secured in its recess by a thin annular ring of epoxy cement. An iron-constantan thermocouple embedded in the calorimeter was used to continuously monitor calorimeter temperature, which was indicated by a recording potentiometer. The copper skin-friction balance floating element was mounted flush with the test surface in circular opening, which provided a 0.002-in. radial clearance between the element and the surrounding plate. A linear variable-differential transformer was used to measure the deflection of the element by friction forces. A second recording potentiometer was used to continuously record the balance output. The balance operating principles are detailed in Ref. 6. Three pressure orifices were connected to mercury manometers to measure static pressure on the test surface. Five iron-constantan thermocouples were embedded just below the test surface along the plate longitudinal centerline to determine plate-temperature distributions.

The first test model had a smooth surface, whereas each of the succeeding three had uniformly rough surfaces consisting of 90° V grooves oriented transversely to the flow direction. The roughness pitch dimensions were 0.005, 0.010, and 0.030 in. (Fig. 1). The 90° groove angle resulted in a groove depth that was approximately one-half of the pitch. Copper, which was selected for the plate body to minimize temperature gradients, proved too soft to machine the V grooves directly into the test surface. The desired roughness was obtained on the plate surface, floating-element, and calorimeter disks by first coating them with thin layers of tin-lead solder into which the grooves were then pressed, using a steel roller having several grooves machined into its periphery. The 2-in.-diam roller was installed in a machine-shop shaper in place of the usual cutting bit to roll the grooves in the test surface. This technique produced a highly uniform test surface free of ragged edges and irregularities. A strip extending back 1 in. from the plate leading edge was left smooth on each model; a one-half-in.-wide tripper strip of No. 80 grit cloth was secured across the rear half of this region.

For all tests, the plate temperature was maintained at approximately 555°R by cooling-water regulation. Variations in heat-transfer rates were obtained by selecting several tunnel stagnation temperatures. For runs during which heat transfer was measured, as flow conditions stabilized, water from a retractable probe was directed to the calorimeter disk in order to depress its temperature below that of the surrounding plate. After probe retraction, the calorimeter temperature increased rapidly toward the flow recovery temperature. The slope of the temperature record, at the instant when the calorimeter and plate temperatures were equal,

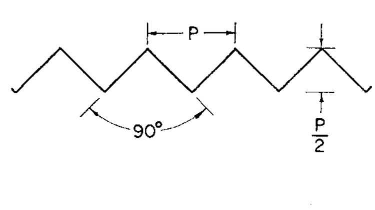

Fig. 1 Cross - section of surface roughness.

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